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DETERMINATION OF THERMOPHYSICAL PROPERTIES OF SEMICONDUCTORS FROM MEASUREMENT OF ETTINGSHAUSEN EFFECT BY METHOD OF VARIATION OF MAGNETIC FIELD

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A method is proposed for determination of the thermophysical coefficients of semiconductors by measurement of the Ettingshausen effect in the unsteady state following the application or removal of a magnetic field.

Investigation of the main galvano- and thermomagnetic effects (GTME) in semiconductors is widely used to obtain information about the electric and magnetic properties of the object [1-4]. The main effects in the specimen are accompanied by superposed thermal effects (Peltier, Ettingshausen, etc.) due to energy transfer by electrons. Lisker [3] proposed an experimental method of determining the kinetic parameters of solids by varying the thermal, electric, and magnetic fields. This enabled him [4] to separate and measure both the main and superposed effects. In view of this it is also possible to determine the thermophysical characteristics of the investigated materials from measurements of the superposed effects. One method, based on the use of the Ettingshausen effect, is examined in this paper. We propose a method of determining the thermal diffusivity k and the thermal conductivity λ from experimental measurements of the Ettingshausen effect in the unsteady state.

The Ettinghausen galvanomagnetic effect is a thermal, i.e., inertial concomitant of the Hall effect, and is characterized by the formation in the specimen of a temperature drop in a direction perpendicular to the current I flowing through it and also to the magnetic field H applied to it. In the steady state the Ettinghausen temperature drop ΔT_E and the heat flux w_E corresponding to it are given by the expressions [2]

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$$\Delta T_E = B(r,\ \mu^*)\, k_0\, (e\lambda d)^{-1} \sigma T R_H I H, \tag{1}$$

$$w_E = B(r,\ \mu^*)\, k_0 e^{-1} \sigma R_H j H T = A T, \tag{2}$$

where the coefficient $B(r, \mu^*)$ depends on the scattering mechanism and the degree of degeneracy of the electron gas.

The steady thermal state of a rectangular specimen with a current, situated in a transverse magnetic field, was considered in [5]. The expressions obtained in [5] can be used to determine the Ettingshausen coefficient with due allowance for heat loss from the surface of the specimen and heat transfer along the conducting wires. The question of determining the thermophysical parameters of the specimen, however, was not considered. The posed problem can be solved by considering the unsteady process of establishment of the thermodynamically equilibrium state in the specimen when a magnetic field is applied to it.

Below we carry out calculations for two cases: 1) the increase in the transverse temperature drop when the magnetic field is applied and; 2) the reduction of the Ettingshausen temperature drop when the magnetic field is removed. We assume here, as in [5], that there is no temperature gradient in the directions x (current) and z (magnetic field), the ambient temperature T_m is constant, the material of the specimen is isotropic, and also for simplicity we ignore the heat loss through the Hall, thermocouple, and current probes, and the Peltier effect.

1. Application of Magnetic Field H at Time $t = 0$. The heat-conduction equation and the boundary conditions have the form:

$$\frac{\partial^2 T'(y,\ t)}{\partial y^2} - \frac{2a}{\lambda d} T'(y,\ t) + \frac{j^2}{\sigma\lambda} = \frac{1}{k} \frac{\partial T'(y,\ t)}{\partial t}, \tag{3}$$

$$\lambda \frac{\partial T'(0,\ t)}{\partial y} = a T'(0,\ t) - w_E, \tag{4}$$

$$-\lambda \frac{\partial T'(b,\ t)}{\partial y} = a T'(b,\ t) + w_E, \tag{5}$$

$$T'(y,\ 0) = 0, \tag{6}$$

where

$$T'(y,\ t) = T(y,\ t) - T_m. \tag{7}$$

Equation (3) takes into account the Joule heat and heat loss from the side faces of the specimen to the air (vacuum). The Ettingshausen effect in this problem is taken into account in boundary conditions (4) and (5) by the additional Ettingshausen heat flux w_E along the y axis. The general heating of the specimen above the ambient temperature due to the current flowing before application of the magnetic field is ignored, since the condition $T = T_m$ can be fulfilled.

The solution is obtained by the Laplace method. For the temperature drop $\Delta\tilde{T} = \tilde{T}(0) - \tilde{T}(b)$, we obtain the expression

$$\Delta\tilde{T}(p) = \frac{2w_{Em}}{\lambda} \frac{\frac{1}{k}(1 - e^{-Bb})^2 + \frac{Ba}{p\lambda}(1 - e^{-2Bb}) + \frac{1}{p}\left(\frac{2a}{\lambda d} + \frac{j^2}{\sigma\lambda T_m}\right)(1 - e^{-Bb})^2}{B\left[\left(B + \frac{a-A}{\lambda}\right)\left(B + \frac{a+A}{\lambda}\right) - \left(B - \frac{a-A}{\lambda}\right)\left(B - \frac{a+A}{\lambda}\right)e^{-2Bb}\right]}, \tag{8}$$

where $B^2 = p/k + 2a/\lambda d$, $w_{Em} = AT_m$ is the Ettingshausen heat flux for a specimen with average temperature T_m.

For the case of large p, which corresponds to the initial (in time) portion of the $\Delta T(t)$ curve, $\exp(-Bb) \to 0$. Omitting in (8) all the terms containing $\exp(-Bb)$ and decomposing the remaining expression into the simplest terms, we obtain

$$\Delta\tilde{T}(p) \approx \frac{h_0}{pP} + \frac{h_-}{p\left(P + (a - A)\sqrt{k}\,\lambda^{-1}\right)} + \frac{h_+}{p\left(P + (a + A)\sqrt{k}\,\lambda^{-1}\right)}, \tag{9}$$

$$P = \left(p + \frac{2ak}{\lambda d}\right)^{1/2}, \quad h_0 = \frac{2j^2 A\sqrt{k}}{\sigma(a^2 - A^2)}, h_\pm = \frac{j^2\sqrt{k}}{\sigma(A \pm a)} + \frac{w_{Em}\sqrt{k}}{\lambda}.$$

Performing the inverse transformation in Eq. (9) we note that the fraction $g_1(p) = [(p + \alpha)^{1/2} + \delta]^{-1}$ is converted [7] through a parabolic cylinder function to a Whittaker function [6]

$$g_1(p) \to f_1(t) = (4\pi\delta)^{-1/2} t^{-3/4} \exp\left[\left(\frac{\delta^2}{2} - \alpha\right) t\right] W_{-3/4,\,1/2}(\delta^2 t). \tag{10}$$

Using the known property of the Laplace transform

$$hp^{-1} g_1(p) \to h \int_0^t f_1(\tau)\, d\tau$$

and the integral representation of the Whittaker function [6], we obtain after double integration

$$\frac{h}{p\,[(p+\alpha)^{1/2} + \delta]} \to \frac{h}{\alpha - \delta^2} \{\alpha^{1/2} - \delta - (\pi t)^{-1/2} e^{-\alpha t} [e^{\frac{\delta^2 t}{2}} D_{-2}(\sqrt{2\delta^2 t}) - e^{\frac{\alpha t}{2}} D_{-2}(\sqrt{2\alpha t})]\}.$$

Taking into account the relation between the parabolic cylinder functions $D_{-2}(z)$ and the probability integral $\Phi(z)$ [6] and collecting together the contributions from all three terms in (9), we obtain a general expression at small t for $\Delta T(t)$, which by expansion in a series of powers of kt is finally brought to the form

$$\Delta T(t) = \frac{4}{\sqrt{\pi}} \frac{w_{Em}}{\lambda} \sqrt{kt}\,(1 - \xi), \tag{11}$$

where $\xi = \frac{\sqrt{\pi}\, a}{2\lambda} \sqrt{kt} + \frac{2}{3}\left(\frac{a}{\lambda d} - \frac{a^2 + A^2}{\lambda^2} + \frac{j^2}{\sigma \lambda T_m}\right) kt + \ldots$.

We now consider the question of the small-time criterion. Expression (11) was obtained on condition that $\exp(-Bb) \ll 1$, which (slightly increasing the inequality) reduces to $p^{-1/2} \ll k^{-1/2} \min\{b, (\lambda d/2a)^{1/2}\}$. Multiplying the inequality by $p^{-\nu}$, where $\nu > 0$ (e.g., $\nu = 1$), we use the inverse Laplace transformation and obtain the small-time condition

$$(kt)^{1/2} \ll \min\left\{b, \left(\frac{\lambda d}{2a}\right)^{1/2}\right\}.$$

Similar conditions for t are imposed in the case of expansion in a series of powers of the probability functions

$$(kt)^{1/2} \ll \min\left\{\left(\frac{\lambda d}{2a}\right)^{1/2}, \frac{\lambda}{|a - A|}, \frac{\lambda}{a + A}\right\}.$$

Combining the inequalities we finally obtain the small t condition:

$$(kt)^{1/2} \ll \min\left\{b, \left(\frac{\lambda d}{2a}\right)^{1/2}, \frac{\lambda}{|a| + |A|}\right\}. \tag{12}$$

To obtain the order of magnitude of the small-time interval we consider a specimen with dimensions $d = 4b = 4$ cm, and coefficients $\lambda = 1.3\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$, $C = 0.5 \cdot 10^3\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}}$, and $\rho = 10^3\ \mathrm{kg \cdot m^{-3}}$. If such a specimen is contained in a vacuum ($a \approx 4\ \mathrm{W \cdot m^{-2} \cdot K^{-1}}$) the small-time interval is determined by the first term in (12) and its value is $t \ll 120$ sec. In a still air chamber a is increased by a factor of ~5, but the first term in (12) remains a minimum; thus, for the considered cases expression (12) can be replaced by the simpler inequality

$$(kt)^{1/2} \ll b. \tag{13}$$

We turn to expression (11), which characterizes the increase in the transverse temperature drop due to the Ettingshausen effect in the initial period of time after application of a constant magnetic field at $t = 0$. It is apparent that in a first approximation the increase conforms to a power law with exponent 1/2. The first correction term takes into account the heat loss from the surface of the specimen and reduces the general increase.

In adiabatic conditions ($a = 0$) the correction ξ begins at the term proportional to kt. The small-time criterion is also simplified and takes the following form:

$$(kt)^{1/2} \ll \min\left\{b, \frac{\lambda}{A}\right\}. \tag{14}$$

In the formulation and solution of the problem in practice we can ignore the slight changes in temperature in expression (2) for the Ettingshausen heat flux w_E, i.e., we can regard $w_E = \text{const}$. This simplification leads to slight changes in the resultant equation (11), viz., to the disappearance of terms of the form $^2/_3 kt\,(A^2/\lambda^2 + j^2/\sigma\lambda T_m)$ in the correction ξ.

2. Removal of Magnetic Field. The problem of the disappearance of ΔT_E when the magnetic field is removed is formulated as follows:

$$\frac{\partial^2 T}{\partial y^2} - \beta^2 (T - T_m) + \frac{j^2}{\sigma\lambda} = \frac{1}{k}\,\frac{\partial T}{\partial t}, \tag{15}$$

$$\lambda\,\frac{\partial T(0,\ t)}{\partial y} - a\,(T(0,\ t) - T_m) = 0, \tag{16}$$

$$\lambda\,\frac{\partial T(b,\ t)}{\partial y} + a\,(T(b,\ t) - T_m) = 0, \tag{17}$$

$$T(y,\ 0) = T_m + G - H_-\exp(\beta y) - H_+\exp(-\beta y), \tag{18}$$

where

$$\beta^2 = \frac{2a}{\lambda d}, \quad G = \frac{j^2 d}{2a\sigma}, \tag{19}$$

$$H_{\mp} = \frac{\exp(\mp b\beta)\,[aG - A\,(G + T_m)](\lambda\beta \mp a \mp A) + [aG + A(T_m + G)](\lambda\beta \pm a \mp A)}{\exp(b\beta)\,[(\lambda\beta + a)^2 - A^2] - \exp(-b\beta)\,[(\lambda\beta - a)^2 - A^2]}. \tag{20}$$

As the initial condition (18) we use the solution of the steady-state problem, in which the temperature drop along the y axis is given by the equation

$$\Delta T_{\mathrm{st}} = (H_+\exp(-b\beta) - H_-)(\exp(b\beta) - 1). \tag{21}$$

The solution of system (15)-(18) is also obtained by the Laplace method. If we confine ourselves (as in the case of application of H) to the initial portion of the decay of $\Delta T(t)$, the expression corresponding to it for $\Delta\tilde{T}(p)$ takes the form

$$\Delta\tilde{T}(p) = \frac{\Delta T_{\mathrm{st}}}{p}\left(1 - \frac{a\sqrt{k}\,\lambda^{-1} + \beta\,\mathrm{cth}\,\dfrac{\beta b}{2}}{a\sqrt{k}\,\lambda^{-1} + B}\right). \tag{22}$$

Performing the inverse Laplace transformation as in (9)-(11) and using inequalities of type (12) we obtain an equation for the decay curve of the Ettingshausen effect:

$$\Delta T(t) = \Delta T_{\mathrm{st}}\left\{1 - \left(\frac{a}{\lambda} + \beta\,\mathrm{cth}\,\frac{\beta b}{2}\right)\left[\frac{2}{\sqrt{\pi}}(kt)^{1/2} - \frac{a}{\lambda}kt - \frac{4a}{3\sqrt{\pi}\lambda}\left(\frac{1}{d} - \frac{a}{\lambda}\right)(kt)^{3/2} + \ldots\right]\right\}. \tag{23}$$

Thus, the decay of ΔT_E conforms in a first approximation to a $1 - C\sqrt{kt}$ law. The equation for the decay of the effect in adiabatic conditions is obtained from (23) with $a = 0$:

$$\Delta T_{\mathrm{ad}}(t) \simeq \Delta T_{\mathrm{st}}\left[1 - \frac{4}{\sqrt{\pi}}\,\frac{\sqrt{kt}}{b} + \ldots\right]. \tag{24}$$

The obtained expressions can be used to determine the thermophysical parameters of semiconducting specimens by the following procedures:

1) from measurements of the decay of the Ettingshausen effect (24) the thermal diffusivity k can be determined. To do this the relation $1 - \Delta T(t)\,(\Delta T_{\mathrm{st}})^{-1} = f(t^{1/2})$ is graphed. Near the origin the graph is a straight line of gradient $\tan\xi$, equal to $4\pi^{-1/2}b^{-1}k^{1/2}$. Then

$$k = \frac{\pi}{16}\,b^2\,\mathrm{tg}^2\,\xi; \tag{25}$$

2) from the known density ρ and specific heat c_V of the substance we can calculate the thermal conductivity λ from the equation

$$\lambda = kc\rho; \tag{26}$$

3) the Ettingshausen heat w_E can be determined from measurements of the initial rise of the Ettingshausen effect (11) from the equation

$$w_E = \frac{\sqrt{\pi}\,\lambda}{4\sqrt{k}}\,\mathrm{tg}\,\theta, \tag{27}$$

where tan θ is the gradient of the straight-line portion of relation $\Delta T(t^{1/2})$ near the origin.

Since expression (2) for w_E contains easily measured quantities, it is also possible to obtain the numerical value of the coefficient $B(r, \mu^*)$, whose value is required for determination of the degree of degeneracy of the gas, the current carriers, and their scattering mechanism.

Thus, we have shown that measurements of the Ettingshausen galvanomagnetic effect is a varying magnetic field can be used to obtain (in addition to determination of the GTM effects themselves) additional information about the thermophysical properties of the investigated materials.

NOTATION

b, width of specimen; d, S, perimeter and area of cross section; I, current strength; j, current density; H, magnetic field strength; T, absolute temperature; T_m, ambient temperature; λ, k, σ, R_H, thermal conductivity, thermal diffusivity, electrical conductivity, and Hall coefficient of investigated material; α, coefficient of heat transfer between specimen and surroundings; $B(r, \mu^*)$, coefficient dependent on scattering mechanism and degree of degeneracy of electron gas; k_0, e, Boltzmann constant and electron charge; $w_E = AT$, Ettingshausen heat released or absorbed on faces of specimen perpendicular to y axis; $\Phi(z)$, probability function; $W_{\mu,\nu}(z)$, Whittaker function; $D_\nu(z)$, parabolic cylinder function; t, time; p, Laplace transform parameter.

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USE OF PERIODIC-HEATING METHOD FOR MEASUREMENT OF THERMAL ACTIVITY COEFFICIENT OF GASES

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A method of increasing the sensitivity of the circuit for measuring the thermal activity coefficient of gases by the periodic-heating method is proposed. The thermal activity of argon, carbon dioxide, and air at 1300°K was measured.

Among the new developing methods of measuring the thermophysical properties of substances the periodic-heating method merits great attention. This method consists essentially in recording the temperature fluctuations of a fast sensor (wire or foil) due to heating by alternating current in the medium under investigation. The method is based on the use of electronic circuitry, is suitable for automation, and can be used to carry out investigations in a wide range of temperature and pressure.

This method was first proposed in [1, 2], where it was used to determine the thermal activity coefficient of liquids (foil sensor) [1, 2]. Later, American authors [3, 4] used practically the same measuring circuit to determine the thermal conductivity of gases by means of wire sensors. This technique was subsequently developed successfully in [5, 6]. Some modifications of the method of measuring the thermal activity coefficients of fluids have been used in a series of investigations [7, 8].

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